ME729 Advanced Robotics - Singularity and Motion Trajectories

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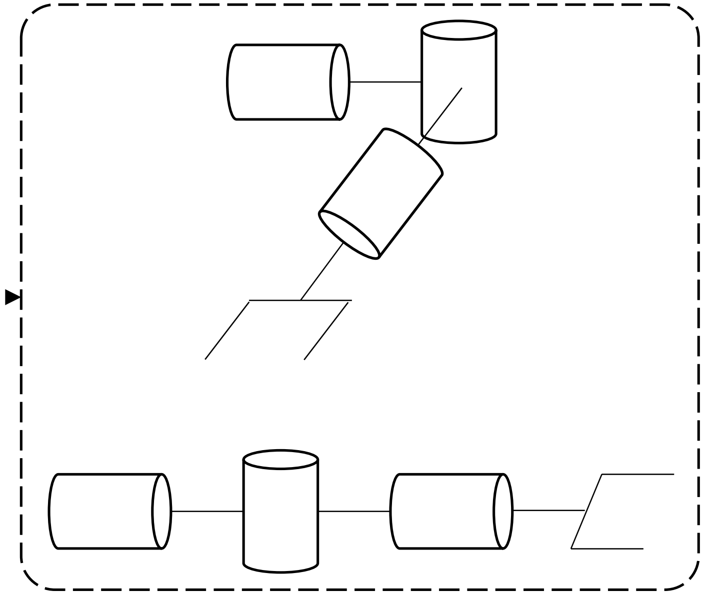
Singularity

❑ Kinematic singularities

- The inverse mapping from Cartesian space to joint space is not sometimes defined.

- Those positions of the robot are referred to as singularities or degeneracies.
- At a singularity,
  - Infinite inverse kinematic solutions may exist.
  - Small Cartesian motions may require infinite joint velocities.
- Boundary Singularities (also known as workspace singularities)
  - Usually caused by a full extension of a joint.
- Internal Singularities (also known as joint space singularities)
  - Caused by an alignment of the robots axes in space.



- **By analyzing the Jacobian matrix of a manipulator**, we can find the singular positions of the robot.

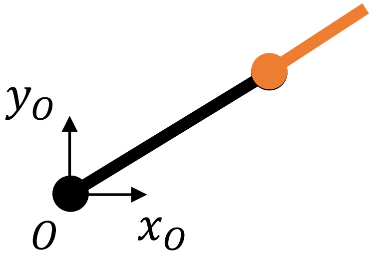
Singularity

- Finding singularities of the 2-link manipulator
  - If we invert the Jacobian, we get $\dot{\boldsymbol{\theta}} = J(\boldsymbol{\theta})^{-1}\dot{\boldsymbol{X}}$.
  - The inverse is undefined whenever $\det(J) = 0$; It is a singular matrix.
  - So, by solving $\det(J) = 0$, we can find singularities in the robot workspace.

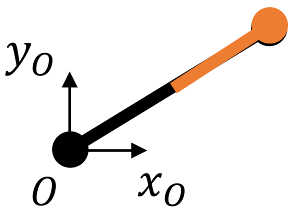
> ❖ To remind formula for determining the inverse of 2x2 matrix.
> If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
> ❖ $\det(A) = ad - bc$.

$$\det(J) = L_1 L_2 s_2$$

  - $L_1 L_2 s_2 = 0$, where L_1 and L_2 are constant.
  - If $s_2 = 0$, the manipulator is at full extension ($\theta_2 = 0$), or looped back onto link 1 ($\theta_2 = 180$).



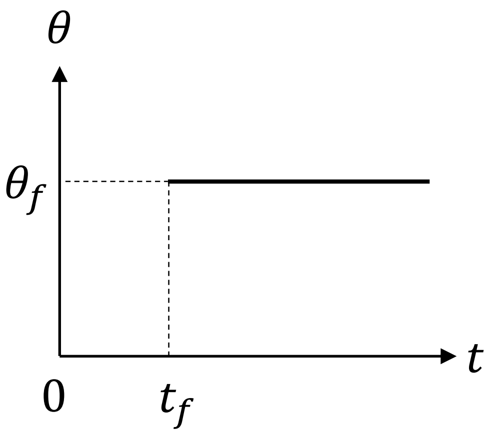
$\theta_2 = 0$



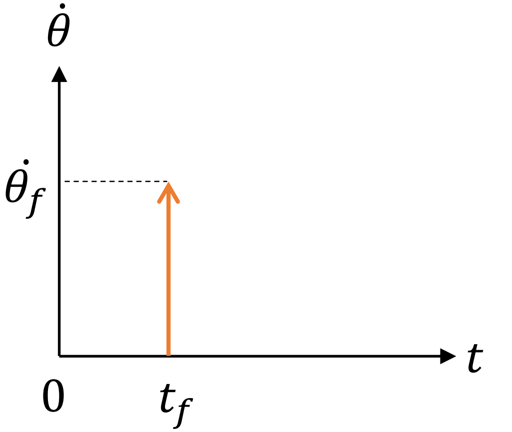
$\theta_2 = 180$

Motion Trajectories

- Why do we need them?
  - A set of desired angles are determined by application of the inverse kinematics.
  - During the last lab, we've just sent desired angles to motors as a reference input.
  - These references are step inputs and have problems.
  - If $\theta = \theta_f$ at t_f, it's a step input and discontinuous at t_f.
  - So, Its velocity becomes impulse.
    - It consumes the high current suddenly.
    - It makes a jerk motion.
    - It makes the system broken.



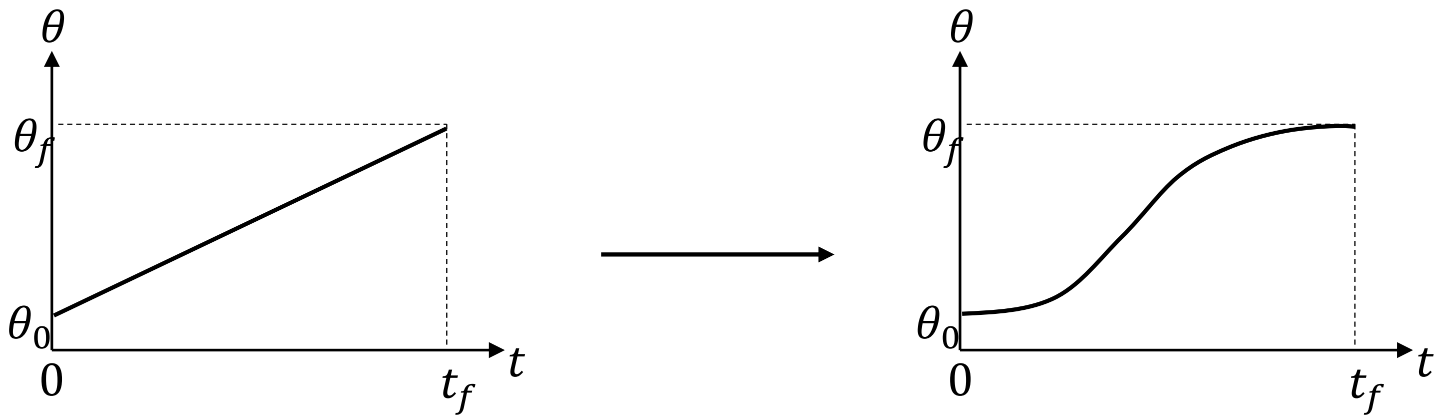
Step input



Impulse velocity

Motion Trajectories

- To make trajectories smooth
  - A simple way is to connect both by a linear function.
    - $\theta(0) = \theta_0, \theta(t_f) = \theta_f$
    - Its velocity also has discontinuous points at boundaries.
  - We should design a trajectory which has ***at least*** zero velocities at boundaries. → Cubic polynomials
    - $\theta(0) = \theta_0, \theta(t_f) = \theta_f$
    - $\dot{\theta}(0) = 0, \dot{\theta}(t_f) = 0$
  - I recommend that a trajectory has zero velocities and accelerations at boundaries. → Quintic polynomials
    - $\theta(0) = \theta_0, \theta(t_f) = \theta_f$
    - $\dot{\theta}(0) = 0, \dot{\theta}(t_f) = 0$
    - $\ddot{\theta}(0) = 0, \ddot{\theta}(t_f) = 0$



Motion Trajectories

❑ Cubic polynomials

- Two constraints on the function's value come from the selection of initial and final values:

$$\theta(0) = \theta_0, \theta(t_f) = \theta_f$$

- An additional two constraints are that the function is continuous in velocity, which in this case means that the initial and final velocity are zero:

$$\dot{\theta}(0) = 0, \dot{\theta}(t_f) = 0$$

- These four constraints can be satisfied by a polynomial of at least third degree.

$$\theta(t) = \underline{a_3} t^3 + \underline{a_2} t^2 + \underline{a_1} t + \underline{a_0}$$

- And the four coefficients are determined by the four constraints.

$$\theta_0 = a_0$$
$$\theta_f = a_3 t_f^3 + a_2 t_f^2 + a_1 t_f + a_0$$
$$0 = a_1$$
$$0 = 3a_3 t_f^2 + 2a_2 t_f + a_1$$

Solving these equations for the a_i, →

$$a_0 = \theta_0$$
$$a_1 = 0$$
$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0)$$
$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0)$$

Motion Trajectories

- Quintic polynomials
  - The first four constraints are same as cubic polynomials' those

$$\theta(0) = \theta_0, \theta(t_f) = \theta_f, \dot{\theta}(0) = 0, \dot{\theta}(t_f) = 0$$

  - An additional two constraints are that the function is continuous in acceleration, which in this case means that the initial and final acceleration are zero:

$$\ddot{\theta}(0) = 0, \ddot{\theta}(t_f) = 0$$

  - These six constraints can be satisfied by a polynomial of fifth degree.

$$\theta(t) = \underline{a_5}t^5 + \underline{a_4}t^4 + \underline{a_3}t^3 + \underline{a_2}t^2 + \underline{a_1}t + \underline{a_0}$$

  - And the all coefficients are determined by the six constraints.

$$\theta_0 = a_0$$
$$\theta_f = a_5 t_f^5 + a_4 t_f^4 + a_3 t_f^3 + a_2 t_f^2 + a_1 t_f + a_0$$
$$0 = a_1$$
$$0 = 5a_5 t_f^4 + 4a_4 t_f^3 + 3a_3 t_f^2 + 2a_2 t_f + a_1$$
$$0 = a_2$$
$$0 = 20a_5 t_f^3 + 12a_4 t_f^2 + 6a_3 t_f + 2a_2$$

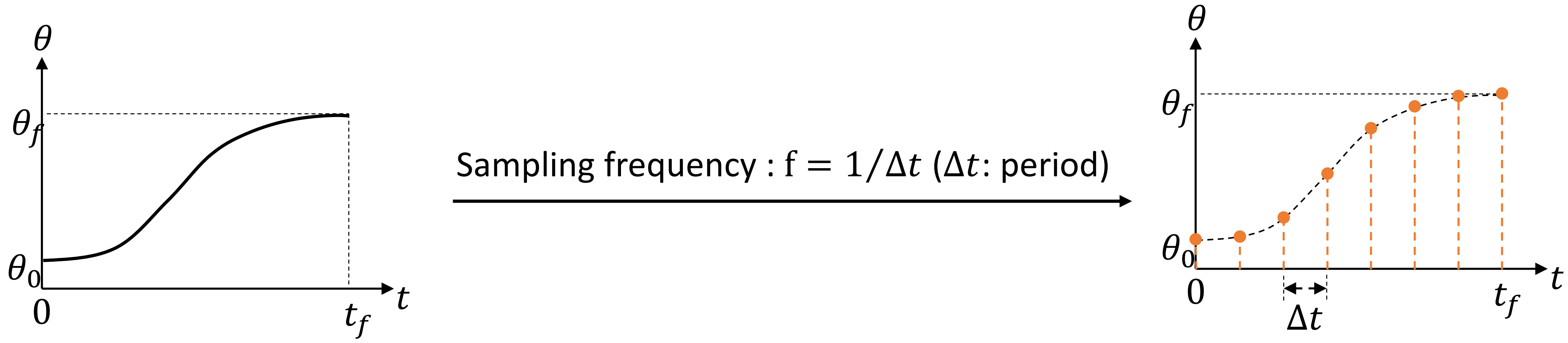
Solving these equations for the a_i, →

$$a_0 = \theta_0, a_1 = 0, a_2 = 0$$
$$a_3 = \frac{10}{t_f^3}(\theta_f - \theta_0)$$
$$a_4 = -\frac{15}{t_f^4}(\theta_f - \theta_0)$$
$$a_5 = \frac{6}{t_f^5}(\theta_f - \theta_0)$$

Motion Trajectories

❑ The way to implement motion trajectories

- We need to discretize continuous trajectories with a sampling frequency.



- $t_i = \Delta t * i$ and $\theta_i = \theta(t_i)$, where $i = 0, \cdots, n$.
- And then, we set the sampled points to reference input sequentially.
- So, we need a timer to discretize and update reference input in real-time.