# Numbering Systems and Digital Electronics

**Objective:** To understand and appreciate the binary number system

1. Quantities can be represented in any base

Example: The number 2157 is actually:

$$2\times10^{3}+1\times10^{2}+5\times10^{1}+7\times10^{0}$$

Or

$$2000+100+50+7=2157$$

Known as Base-10 system (decimal). Perhaps evolved because people have 10 fingers

2. Computers use a Base-2 system (binary)

3. Bits represent on-off switches, 1 and 0 respectively

Example: The (binary) number 10011 (to a computer) is actually:

$$1\times2^{4}+0\times2^{3}+0\times2^{2}+1\times2^{1}+1\times2^{0}$$

Or

$$16+0+0+2+1=19$$

This process is called a binary-to-decimal conversion

## 4. Decimal-to-binary conversion

Example: Convert $234_{10}$ to binary

Step 1: Form table

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| | | | | | | | |

(1)

Step 2: Populate table

$234_{10}$ =

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |

Sanity Check: $$128+64+32+0+8+0+2+0=234$$

Answer: $$234_{10}=11101010_2$$

## 5. n-bit machine

Largest decimal number represented by n-bits: $2^n - 1$

Example:

$255_{10}$ =

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

n = 8 bits, $2^8$-1 = 255

$256_{10}$ =

| $2^8$ | $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|---|
| 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$256_{10}$ appears to be $0_{10}$ on 8-bit machines.

This is called overflow

6. C Programming: `int` is a 16-bit number (2 bytes)

$n = 16$ bits, $2^{16}-1 = 65535$

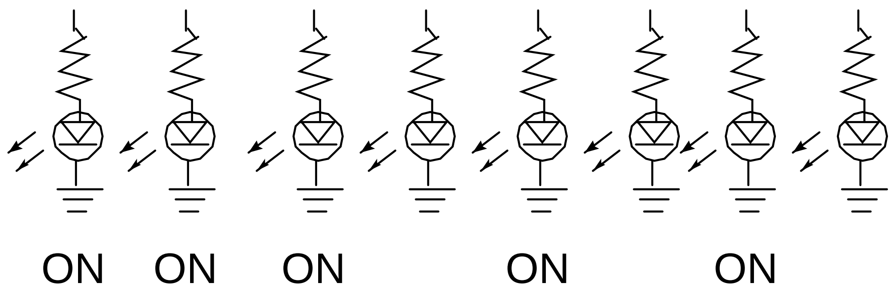
7. Digital Electronics: +5 Volts = $1_2$ and GND = $0_2$

Example:

$234_{10} =$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |

Digital Output Port
(e.g. parallel or I2C)




8. Turning on LEDs is a big deal