Bilateral Controller for Teleoperators with Time Delay via μ-Synthesis

Gary M. H. Leung, Bruce A. Francis, *Fellow, IEEE,* and Jacob Apkarian

***Abstract*—In the standard teleoperator system, force and velocity signals are communicated between a master robot and a slave robot. It is well known that the system can become unstable when even a small time delay exists in the communication channel. In this paper, a method based on the $\mathcal{H}_\infty$-optimal control and μ-synthesis frameworks is introduced to design a controller for the teleoperator that achieves stability for a prespecified time-delay margin while optimizing performance specifications. A numerical design example is included.**

I. Introduction

TELEOPERATION is the extension of a person's sensing and manipulation capability to a remote location [18]. We consider the standard teleoperator system depicted in Fig. 1. The human operator commands via a master manipulator by exerting a force f_h. The master moves with velocity v_m that is transmitted via the communication network to the slave manipulator. Likewise, the slave manipulator responds to the reference signal, v_{sd}, and the force f_s sensed as a result of contact with the environment and/or some external source, f_e, is transmitted back to the master network, which results in the force f_{md} to the master. In general, all of these signals can be vector-valued functions of continuous time.

In 1965, Ferrell [8] first showed the instability due to time delay of a kinesthetically coupled teleoperator. Vertut *et al.* [20] showed that stability could be achieved in the presence of time delay only when the bandwidth of the system was severely reduced. However, Brooks [6] suggested that the bandwidth of a teleoperator should be approximately between 4 Hz to 10 Hz for normal operation. Hence, the problem of achieving adequate bandwidth in the face of appreciable time delay had remained largely unsolved. Much of the work on time delay in teleoperation has been in the area of human performance. Only a few papers have dealt with the control issues (e.g., input/output stability, controller design) pertaining to this problem.

In 1988, Anderson and Spong [2] introduced a new communication architecture for telerobots that achieves, under certain assumptions, infinite time-delay stability margin in the communication channel. Their idea is as follows. In Fig. 1 set $f_h = f_e = 0$ and assume the master system (f_{md} to v_m) and slave system (v_{sd} to f_s) are strictly passive. Now reconfigure as Fig. 2. Then the new master system ($\tilde{f}_{md}$ to $\tilde{v}_m$) and new slave system ($\tilde{v}_{sd}$ to $\tilde{f}_s$) are strictly contractive ($\mathcal{H}_\infty$-norms strictly less than 1), so by the small-gain theorem the overall system will be stable for any time delay in the communication channel. Of course, this method works only if the original master and slave systems are strictly passive, which then become design constraints for the individual master and slave controllers. How to design with these constraints is not treated in [2]. (It is interesting to note that the discretization, via sample and hold, of a passive system is in general not passive [15]. So the Anderson-Spong method will in general not be directly applicable for digital controllers and communication channel.) In [3], asymptotic stability of master and slave velocities was proven.

In [16] the work in [2], [3] is interpreted using transmitting wave variables in place of the standard power variables (e.g, forces and velocities). In doing so, [16] formulates the input/output variables of the telerobot as two sets of waves of the master and slave networks. The concept of characteristic impedance of these waves is introduced and an analogy is drawn to the characteristic impedance of a natural wave guide. By using this analogy, it is shown that, in order to achieve good performance, the termination of each port of the telerobot network should be chosen to provide damping equivalent to the characteristic impedance of the wave variables.

In [1] a synthesis method to design generalized bilateral control is presented. The method is based on the passivity concept and uses two criteria, "transparency distance" and "passivity distance," to evaluate the design. The case of rigid manipulators is solved. For the case of flexible joint manipulators, the authors advocate the use of $\mathcal{H}_\infty$ control theory to address the problem. In addition, it is stated that "time delay communication is a fundamental performance limitation and that no causal controller can improve the performance."

In [19], and subsequently [14], $\mathcal{H}_\infty$ optimal control theory was applied to the design of a controller for a telerobot. In this design, an ideal communication channel (i.e., no time delay) was assumed and the performance specifications were given in terms of desired dynamic relationships between the master and slave robots. Model reduction techniques were employed and experiments were conducted using the reduced-order controller.

None of the above papers solves the problem of robust stability and performance of the teleoperator in the presence of time delay in the communication channel. In many cases,

Manuscript received April 13, 1993; revised September 28, 1993. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

G. M. H. Leung is with Allied Signal Aerospace Canada, Etobicoke, Ontario, M9W 6L7 Canada.

B. A. Francis is with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, M5S 1A4 Canada.

Jacob Apkarian is with Quanser Consulting, Toronto, M5T 2N7 Canada.

IEEE Log Number 9403330.

Fig. 1. Standard teleoperator system.

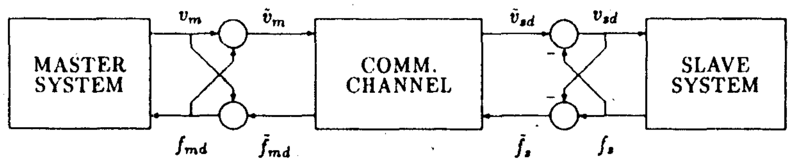


Fig. 2. Design of Anderson and Spong.

the controller design is specific to the case where the slave manipulator is in contact with the environment. That is, it is assumed that the slave manipulator is always in contact with its environment and little or no consideration is given to the case where the slave manipulator moves in free space. In other cases (e.g., the hybrid formalism [13], [12], [10], [11], [5]) internal stability is not an explicit specification — presumably, one checks for internal stability of the system at the end of the design process.

In this paper, we introduce a method for the design of a controller for a bilateral force-reflecting teleoperator with time delay through the communication channel. This design method is based on $\mathcal{H}_\infty$ optimal control ([7], [9]) and μ-analysis and synthesis techniques [4]. One advantage of our proposed method is that it allows us to design both for performance (time-domain specifications) and time-delay stability margin in one step. In previous works, controllers were designed to satisfy certain nominal performance specifications and then modified to ensure stability for time delay. These modifications were conservative in that although infinite time-delay stability margin was achieved, system performance in the face of a time delay was not a design specification. A second advantage of our method is that, with the control architecture chosen, performance is treated both for free motion and for constrained motion when the slave is in contact with its environment.

We begin by giving an overview of teleoperation and the associated control problem. Next we give a brief introduction to $\mathcal{H}_\infty$ optimal control and the μ-analysis and synthesis technique. We then outline the design procedure and illustrate our results via a design example. In order to keep the presentation clear, we deal only with a single degree-of-freedom linear time-invariant teleoperator model.

The notation used in this paper is that $P(s)$, $T(s)$, $\cdots$ denote transfer functions and $u(s)$, $v(s)$, $\cdots$ denote signal transforms.

A. Design Methodology

The design methodology developed in this paper can be summarized as follows.

The design procedure has two steps. The first is the design for free motion, where the master and slave manipulators move freely and are unconstrained by the environment or any external source (i.e., the case where $f_e = 0$). For this step, we suppose there are two goals: 1) a desired master compliance and 2) motion following of the master by the slave. Separate master and slave controllers are designed and communication is from the master to the slave only. The second step is the design for constrained motion, where the slave manipulator is in contact with its environment and a contact force is sensed at the slave (i.e., the case where $f_e \neq 0$). For this step, we suppose that the goal is that any forces sensed by the slave manipulator must be reflected back to the master manipulator. (Of course, the constraint on this second step is that it should not affect the results of the first step.) In this step an "outer loop" controller is designed, communicating signals from the slave back to the master. Time delay is modeled and a pre-specified time-delay stability margin is achieved.

As a result, the telerobotic system is designed to function both in free motion and when the slave is in contact with its environment, and also when there is a pre-specified upper bound on the communication time delay.

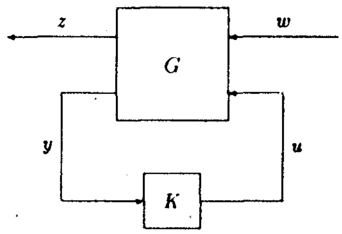


Fig. 3. Standard $\mathcal{H}_\infty$ design setup.

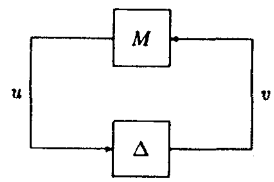


Fig. 4. Feedback interpretation of $\mu_\Delta(M)$.

II. The $\mathcal{H}_\infty$ and μ-Synthesis Design Framework

In this section, we will briefly describe the $\mathcal{H}_\infty$-optimal control framework and the μ-synthesis design framework. Readers familiar with this material may choose to skip this section.

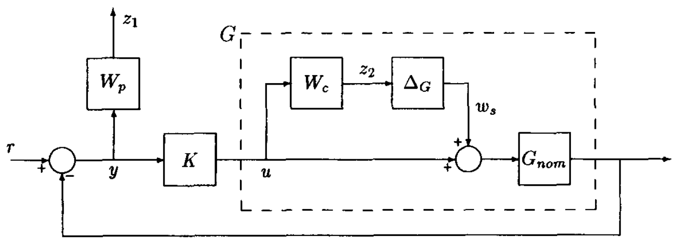


Fig. 5. Standard single-input, single-output system.

A. The $\mathcal{H}_\infty$ Design Framework

The standard setup for the $\mathcal{H}_\infty$ design problem is shown in Fig. 3 [9]. In this figure, w, u, z, and y are vector-valued signals: w is the exogenous input (command/reference signals, disturbances, noises); u is the control signal; z is the output to be controlled and y is the measured output (input to the controller).

In the setup of Fig. 3, G represents a generalized plant (which includes all system dynamics, weighting matrices, etc.) and K represents a controller to be designed. Partition G as

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

where

$$\begin{aligned} z &= G_{11}w + G_{12}u \\ y &= G_{21}w + G_{22}u. \end{aligned} \tag{1}$$

The resulting system from w to z, namely

$$G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}$$

is a linear-fractional transformation of K that depends on G. The notation for this system is $F_l(G, K)$, subscript l indicating that K is connected in the lower loop. Define the $\mathcal{H}_\infty$ norm of a transfer matrix $F(j\omega)$ as

$$\|F\|_\infty := \sup_\omega \bar{\sigma}[F(j\omega)]$$

where $\bar{\sigma}$ denotes the largest singular value.

The standard problem in $\mathcal{H}_\infty$ optimal control is to find a real-rational, proper controller K to minimize $\|F_l(G, K)\|_\infty$ under the constraint of internal stability. The solution requires several assumptions, regarded as *regularity conditions*. For example, the input u needs to be fully weighted in the output z—otherwise the problem is singular and the optimal controller K may be improper; similarly, there need to be sensor noises on all measured signals. Modifying a problem so that the regularity conditions are satisfied is called *regularization*. Solving the standard problem, therefore, amounts to choosing weights on error signals and regularizing. State-space formulae for solving this problem may be found in [7]. The $\mathcal{H}_\infty$ design technique may be performed using the μ-Synthesis and Analysis Toolbox [4] in MATLAB.

B. μ-Analysis and Synthesis

For completeness, we give a brief introduction to the μ-analysis and synthesis technique. For a detailed treatment, see [4].

The issue is structured uncertainty. The structured singular value, a matrix function denoted by $\mu(\cdot)$, is defined as follows. Let $M \in \mathbf{C}^{n\times n}$, S and F be two non-negative integers and $r_1, \cdots, r_S$; $m_1, \cdots, m_F$ be positive integers where

$$\sum_{i=1}^{S} r_i + \sum_{j=1}^{F} m_j = n.$$

Define a family $\mathbf{\Delta} \subset \mathbf{C}^{n\times n}$ of block-diagonal perturbation matrices

$$\begin{aligned} \mathbf{\Delta} = \{&diag[\delta_1 I_{r_1}, \cdots, \delta_S I_{r_S}, \Delta_1, \cdots, \Delta_F] : \\ &\delta_i \in \mathbf{C}, \Delta_j \in \mathbf{C}^{m_j \times m_j}\}. \end{aligned}$$

Thus, a matrix in $\mathbf{\Delta}$ is block diagonal with S scalar blocks and F full blocks. Then $\mu(M)$ is defined to be the reciprocal of the smallest γ such that $I - M\Delta$ is singular for some $\Delta \in \mathbf{\Delta}$, $\bar{\sigma}(\Delta) < \gamma$. We may interpret this definition in the following way. Suppose $M \in \mathbf{C}^{n\times n}$ and consider the loop shown in Fig. 4. Then $\mu(M)$ is a measure of the smallest structured perturbation, Δ, that causes instability of the feedback loop.

To illustrate the μ-synthesis technique, introduce the standard unity feedback system shown in Fig. 5. The dashed box represents the "true" model of the plant with the associated transfer function G. Inside this box is the nominal model of the plant dynamics G_{nom} with the two elements W_c and Δ_G, which parametrize the uncertainty in the model. The weighting function W_c is assumed known and reflects the amount of uncertainty (frequency dependent) in the model. The perturbation Δ_G is unknown but is assumed to be stable with $\|\Delta_G\|_\infty < 1$. The family of plants associated with this uncertainty model is therefore

$$\mathcal{G} := \{G_{nom}(I + W_c\Delta_G) : \Delta_G \; stable, \|\Delta_G\|_\infty < 1\}.$$

Good performance is taken to mean small tracking error. The weighting function W_p reflects the relative importance of various frequency ranges for which performance is desirable. The closed-loop system is said to achieve "nominal performance" if

$$\|W_p(I + G_{nom}K)^{-1}\|_\infty \leq 1.$$

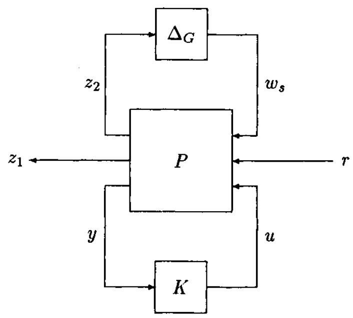


Fig. 6. μ-analysis configuration.

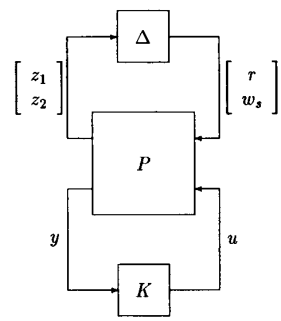


Fig. 7. μ-analysis configuration.

The closed-loop system achieves "robust stability" if it is internally stable for all $G \in \mathcal{G}$, equivalently,

$$\|W_c K G_{nom}(I + K G_{nom})^{-1}\|_\infty \leq 1.$$

Finally, the closed-loop system achieves "robust performance" if it achieves robust stability and, in addition, the performance objective

$$\|W_p(I + GK)^{-1}\|_\infty \leq 1$$

is satisfied for all $G \in \mathcal{G}$.

To handle this problem by the μ-synthesis design technique, let $P(s)$ denote the three input, three output open loop system (i.e., Fig. 5 without controller K and removing the perturbation Δ_G). Then

$$\begin{aligned} P &:= \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \\ &= \begin{bmatrix} \begin{bmatrix} W_p & -W_p G_{nom} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} -W_p G_{nom} \\ W_c \end{bmatrix} \\ [I \quad -G_{nom}] & -G_{nom} \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} z &= P_{11} w + P_{12} u \\ y &= P_{21} w + P_{22} u \end{aligned}$$

where (see Fig. 5)

$$z := \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad w := \begin{bmatrix} r \\ w_s \end{bmatrix}.$$

Redraw the system of Fig. 5 in the standard μ-synthesis configuration of Fig. 6. The problem reduces to finding a controller K to make the $\mathcal{H}_\infty$ norm of the transfer matrix from r to z_1 as small as possible for all allowable Δ_G with the additional constraint of robust stability. This robust performance problem is transformed into the robust stability problem of Fig. 7. The structured perturbation block is

$$\Delta := \begin{bmatrix} \Delta_p & 0 \\ 0 & \Delta_G \end{bmatrix}$$

where Δ_p is a fictitious uncertainty block used to incorporate the $\mathcal{H}_\infty$ performance objective

$$\|W_p(I + GK)^{-1}\|_\infty \leq 1 \quad \forall G \in \mathcal{G}.$$

Using the main loop theorem in [4], a nominally stabilizing controller K achieves robust stability in Fig. 7 iff $\mu[F_l(P, K)j\omega] \leq 1$ for all ω.

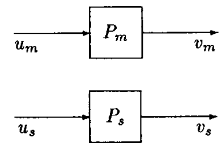


Fig. 8. Master and slave manipulators.

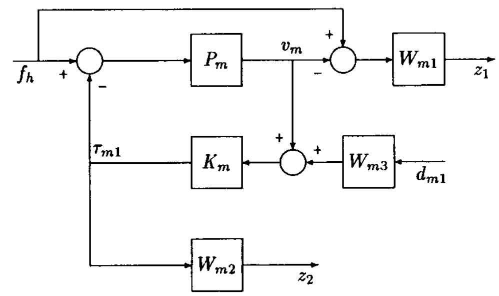


Fig. 9. Design of K_m.

III. Design for Free Motion

Represent the transfer matrix of the master manipulator by $P_m(s)$ and the slave manipulator by $P_s(s)$. The systems can be represented as in Fig. 8, where u_m and u_s are the applied forces on the master and slave manipulators and v_m and v_s are the resultant velocities.

Consider $\mathcal{H}_\infty$-optimal design of controllers for the master and slave manipulators for free motion. Let K_m and K_s denote free motion controllers for the master and slave, respectively. For the master controller the design specifications are taken to be as follows:[1]

1) A desired master compliance should be achieved. In general, this would mean that v_m should be a desired function of f_h, say, $v_m = \Gamma_m f_h$ (Γ_m is some linear time-invariant system). The compliance error would then be $v_m - \Gamma_m f_h$ and this should therefore be included as a component in the vector z to be controlled. For

[1] It will be clear from the development that the general method is not at all restricted to these particular specs; they are merely typical.

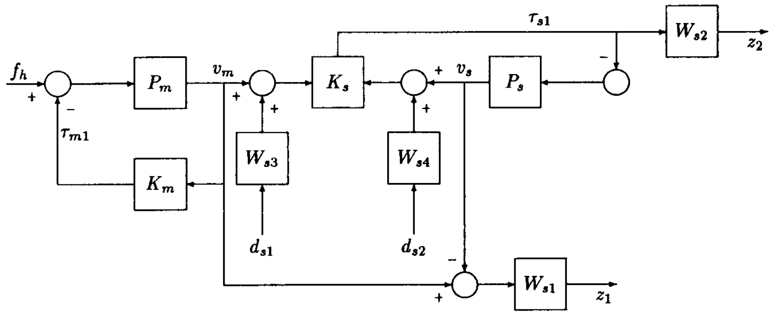


Fig. 10. Design of K_s.

simplicity, we assume $\Gamma_m = I$, so $v_m - f_h$ should be included in z.

2) The plant input, τ_{m1}, should not exceed a prespecified saturation limit, so τ_{m1} should be included in z.

In order to regularize the problem, a noise d_{m1} is included. The $\mathcal{H}_\infty$ design of K_m may be carried out with reference to Fig. 9, where W_{m1}, W_{m2} and W_{m3} are frequency dependent weighting matrices. Thus

$$z = \begin{bmatrix} W_{m1}(f_h - v_m) \\ W_{m2}\tau_{m1} \end{bmatrix}, \quad w = \begin{bmatrix} f_h \\ d_{m1} \end{bmatrix}$$
$$y = v_m + W_{m3}d_{m1}, \qquad u = \tau_{m1}. \tag{2}$$

Transfer functions relating the variables z and y to w and u in Fig. 9 may be determined (as per (1)) and the $\mathcal{H}_\infty$ design procedure may be carried out using MATLAB.

For the slave controller, the design specifications are taken to be as follows:

1) The slave velocity, v_s, should track the master velocity, v_m (so $v_m - v_s$ should be included in z).
2) The control torque, τ_{s1}, should not exceed a prespecified saturation limit (so τ_{s1} should be included in z).

Noises d_{s1} and d_{s2} are introduced to regularize the problem. The $\mathcal{H}_\infty$ design of K_s may be carried out as per Fig. 10. Thus

$$z = \begin{bmatrix} W_{s1}(v_m - v_s) \\ W_{s2}\tau_{s1} \end{bmatrix}, \quad w = \begin{bmatrix} f_h \\ d_{s1} \\ d_{s2} \end{bmatrix}$$
$$y = \begin{bmatrix} v_m + W_{s3}d_{s1} \\ v_s + W_{s4}d_{s2} \end{bmatrix}, \qquad u = \tau_{s1}. \tag{3}$$

Again, W_{s1}, W_{s2}, W_{s3} and W_{s4} are frequency dependent weighting matrices. Note that the design and implementation of K_s does not affect the performance of the master system (i.e., P_m and K_m). This is due to the fact that, for operation in free space (no contact with external environment), there is no feedback signal from the slave system to the master system.

In the design procedure described above, we have not considered the issue of time delay in the transmission of signals from the master to the slave. Although such a time delay does not affect the stability of the system, it nevertheless does degrade performance. A few comments are necessary here. Note that, by choosing to ignore the time delay in our design procedure, we have, in fact, chosen to minimizes the error signal

$$e_1(t) = v_m(t - \tau) - v_s(t)$$

where τ is the transmission time delay from master to slave. However, the true error signal is

$$e_2(t) = v_m(t) - v_s(t).$$

Therefore, one may argue that a more appropriate design methodology would incorporate the time delay element into the weighted plant, by treating it as a perturbation, before carrying out the $\mathcal{H}_\infty$ design. In doing so, robust performance for time delay would be achieved. On the other hand, since $\mathcal{H}_\infty$ optimal control minimizes the worse-case $\mathcal{L}_2$ norm of the error signal over the class of all $\mathcal{L}_2$ input signals, if we choose $e_2(t)$ as our error signal we will likely witness a decline in nominal performance (i.e., when there is no time delay or, more importantly, when the input to the system is far from "worse case"— this is borne out by simulation). Clearly, a choice must be made here. If e_1 is chosen as the error signal, then this choice must be made with the assumption that the operator is aware of the time delay in the system (i.e., the operator applies only inputs to the system that have frequency much less than $\frac{1}{\tau}$). Conversely, if e_2 is chosen as the error signal, then the assumption is that a larger class of inputs to the system is expected (e.g., if the operator is unaware of the transmission time delay and attempts to induce rapid movements). Note that if the latter design specification is chosen, we must incorporate into the design a perturbation model that covers the time delay. The procedure for this is identical to that presented in the following section for constrained motion design.

IV. Design for Constrained Motion

We now consider the design for constrained motion. The overall system is shown in Fig. 11, where the new controller K_c is now to be designed. In this figure, Z_e denotes the (assumed known) impedance of the slave environment ($f_e = Z_e v_s$), K_m and K_s have been designed to satisfy performance specifications for free motion, and f_b is a sensor noise to regularize the problem.

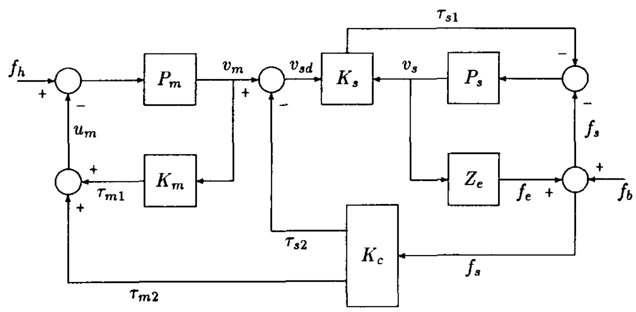


Fig. 11. Design of a controller for constrained motion.

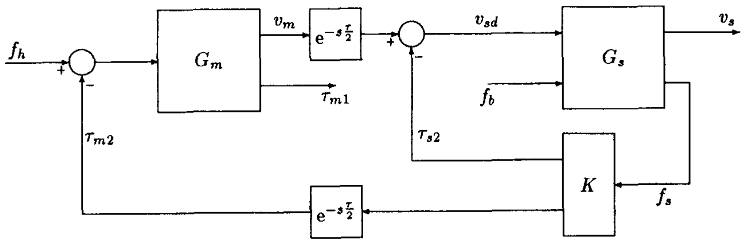


Fig. 12. Master/slave system with time delay.

The design specifications are taken to be as follows:

1) The design of K_c should not affect the free motion already designed for.
2) The velocity error signal is minimized (so $v_m - v_s$ should be included in z).
3) The feedback force of the master hand controller should track the sensed force at the slave end-effector (so $f_s - (\tau_{m1} + \tau_{m2})$ should be included in z).
4) The control torques should not exceed prespecified saturation limits (so τ_{s2} and τ_{m2} should be included in z).
5) The system should be stable for up to a pre-specified amount of time delay through the communication channel between the master and the slave.

The first specification places a constraint on our control system architecture. The extra control loop that is added to the system (of Fig. 10) must not interfere with the operation of the original system when there is no sensed force at the slave end-effector (i.e., when $f_e = 0$). We can satisfy this constraint if we use f_e as the only input to the controller to be designed. This is because, for the case $f_e = 0$ (free space motion), the output of the controller K_c is also zero and hence, will not affect the operation of the system designed for operation in free space. The second, third, and fourth specifications are performance specifications that are similar to that of a normal $\mathcal{H}_\infty$-optimal design. In order to satisfy the fifth specification, we model the time delay in the loop as a perturbation to the system and design the system to be robust to such a perturbation. To design for all specifications in one step without being too conservative, we must use μ-synthesis. We will model the second, third and fourth specifications as one fictitious perturbation block and the fifth specification as a separate perturbation block.

A. The Design Technique

To simplify the discussion, introduce the following notation:

$$\begin{aligned} G_m &:= \begin{bmatrix} (I + P_m K_m)^{-1} P_m \\ K_m (I + P_m K_m)^{-1} P_m \end{bmatrix} \\ &=: \begin{bmatrix} G_{m1} \\ G_{m2} \end{bmatrix} \\ \Gamma_s &:= [I + P_s(K_{s2} + Z_e)]^{-1} \\ K_s &=: [K_{s1} K_{s2}] \\ K_c &=: \begin{bmatrix} K_{cs} \\ K_{cm} \end{bmatrix}. \end{aligned}$$

Also, define

$$\begin{aligned} G_s &:= \begin{bmatrix} -\Gamma_s P_s K_{s1} & -\Gamma_s P_s \\ -Z_e \Gamma_s P_s K_{s1} & (I - Z_e \Gamma_s P_s) \end{bmatrix} \\ &=: \begin{bmatrix} G_{s11} & G_{s12} \\ G_{s21} & G_{s22} \end{bmatrix}. \end{aligned}$$

Then the system of Fig. 11 may be redrawn as Fig. 12, where in addition blocks representing time delay in the communication channel have been included. Time delay of $\frac{T}{2}$ seconds in the transmission of data from the master to the slave and vice-versa is assumed.

It is convenient to lump the delays into one block. So move the lower delay block around the loop to the forward path of the loop and redefine f_b and v_s as

$$f_b \leftarrow e^{-s\frac{T}{2}} f_b \qquad v_s \leftarrow e^{-s\frac{T}{2}} v_s.$$

This does not change the input/output characteristics of the system, that is, the $\mathcal{H}_\infty$ norm of the system is unchanged, since the $\mathcal{H}_\infty$ norm of $e^{-s\frac{T}{2}}$ equals 1. In this way we may perform our design on the system in Fig. 13.

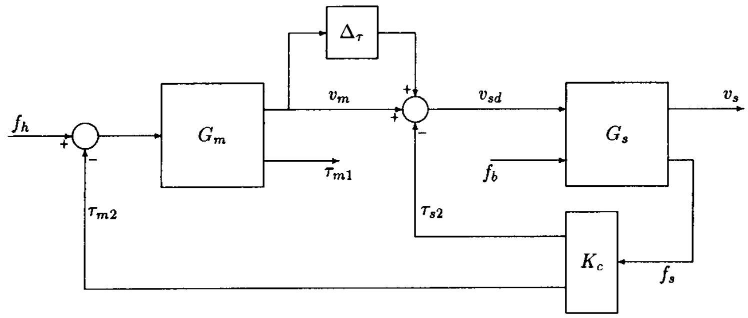


Fig. 13. Master/slave system with time delay.

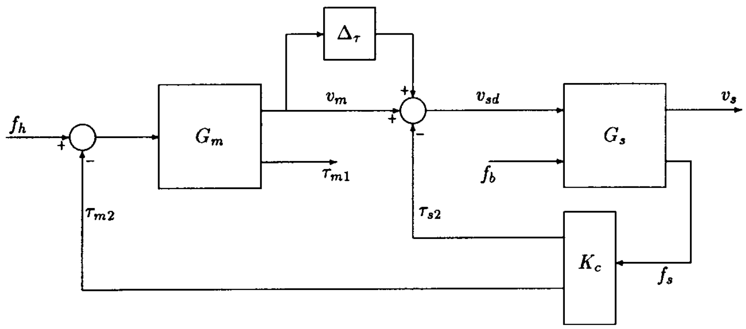


Fig. 14. Time delay as a perturbation.

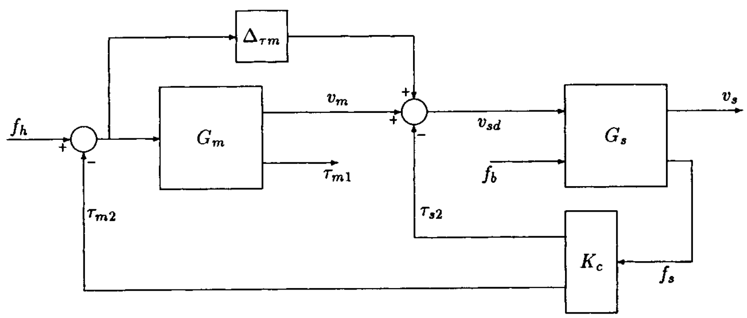


Fig. 15. Alternate perturbation model.

The block $e^{-s\tau}$ represents a time delay (master to slave to master) of τ seconds. We can reconfigure the system of Fig. 13 so that the time delay is reflected as a perturbation to the system. Let

$$\Delta_\tau(s) := e^{-s\tau} - 1.$$

Then the system of Fig. 13 can be redrawn as in Fig. 14. Notice that the maximum magnitude of $e^{-j\omega\tau} - 1$ equals 2, that is, $\|\Delta_\tau\|_\infty = 2$, for every $\tau > 0$, so this perturbation is not small! If we were to design for a perturbation this large, our design would be very conservative indeed, since it would be compensating for all perturbations of norm ≤ 2, not just the time delay.

Observe that $\Delta_\tau(s)$ is small at low frequencies, that is, for every lowpass filter F, $\|\Delta_\tau F\|_\infty \to 0$ as $\tau \to 0$. This suggests that a sensible way to reduce this conservatism is to move the perturbation to surround the master system, G_m, as in Fig. 15. Note that the system of Fig. 15 is equivalent to that of Fig. 14 if $\Delta_{\tau m} = \Delta_\tau G_{m1}$. Since $G_{m1}(s)$ is strictly proper by design, there is a single-input, single-output bandpass filter $W_c(s)$ such that

$$\|W_c{}^{-1}\Delta_{\tau m}\|_\infty < 1. \tag{4}$$

Write $\Delta_{\tau m} = \Delta_{\tau m} W_c^{-1} W_c$ to get Fig. 16. The perturbation is now $W_c{}^{-1}\Delta_{\tau m}$, and it has norm < 1.

In Fig. 16, performance weights W_1, W_2 have been shown, but the controller K and weights W_3, W_4 on the control inputs τ_{m2} and τ_{s2} have not been shown to avoid clutter. Thus, for this design,

$$z = \begin{bmatrix} W_1(v_m - v_s) \\ W_2[f_s - (\tau_{m1} + \tau_{m2})] \\ W_3\tau_{m2} \\ W_4\tau_{s2} \\ z_5 \end{bmatrix}, \quad w = \begin{bmatrix} f_h \\ f_b \\ w_3 \end{bmatrix}$$

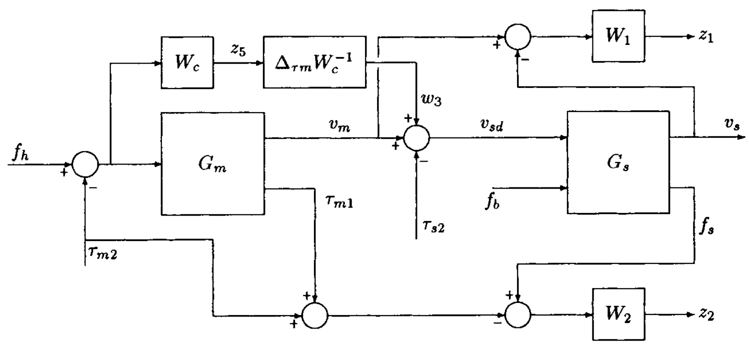


Fig. 16. Design for a constrained motion.

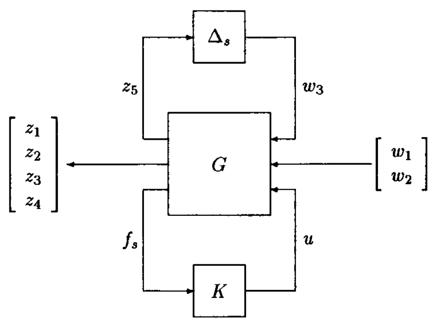


Fig. 17. Preliminary setup for μ-synthesis.

$$y = f_s, \qquad u = \begin{bmatrix} \tau_{m2} \\ \tau_{s2} \end{bmatrix}. \tag{5}$$

This design problem is more clearly illustrated in Fig. 17, where $W_c{}^{-1}\Delta_{\tau m}$ has been replaced by Δ_s. The goal now is to design a controller that achieves the performance specifications for all $\|\Delta_s\|_\infty < 1$. Introducing a fictitious performance perturbation Δ_p (as discussed in Section II), let

$$\Delta := \begin{bmatrix} \Delta_p & 0 \\ 0 & \Delta_s \end{bmatrix}$$

to get the final setup of Fig. 18. The μ-synthesis problem is to find a controller K to minimize $\mu(F_l(G, K))$ in Fig. 18. By appropriate design of weighting matrices, it is possible to design a controller that makes the norm from w_3 to z_5 less than or equal to 1 and, at the same time, makes the norm from the remaining inputs to the remaining outputs

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \rightarrow \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

as small as possible (therefore optimizing performance as well). Note that in our design the size of $\|\Delta_s\|_\infty$ is fixed as this is a hard constraint (stability); the size of $\|\Delta_p\|$ is not fixed.

V. Example

In this section, we present a design example to illustrate the method. To keep the analysis fairly simple, we consider the dynamic model of one degree of freedom master and slave manipulators. Let the master robot be modeled as a one link rigid manipulator:

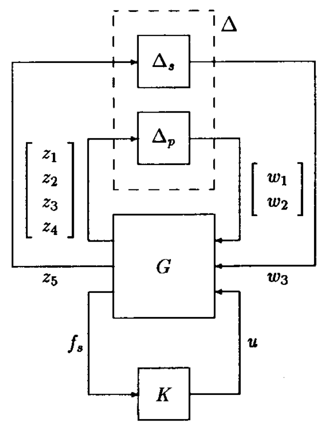


Fig. 18. Final setup for μ-synthesis.

$$v_m(s) = \frac{1}{s}[f_h(s) - \tau_m(s)].$$

For the slave manipulator, we choose a fourth order model of a one link manipulator with two flexible modes. This model is taken from the Shuttle Remote Manipulator System shoulder joint [17] and is described by the following equation:

$$v(s) = -\frac{282s^2 + 1 \times 10^4}{s^4 + 1.25s^3 + 1100s^2 + 1290s + 890}[f_c(s) + \tau_s(s)].$$

It incorporates models of a DC motor, tachometer and linearized geartrain. We now proceed to design controllers for this system for both free motion and for constrained motion. In the discussion below, α_{mi}, α_{si} and α_i are all non-negative real numbers.

A. Design for Free Motion

For this design, the vector-valued signals w, u, z and y in the standard $\mathcal{H}_\infty$ design setup were defined as in (2) (see Fig.

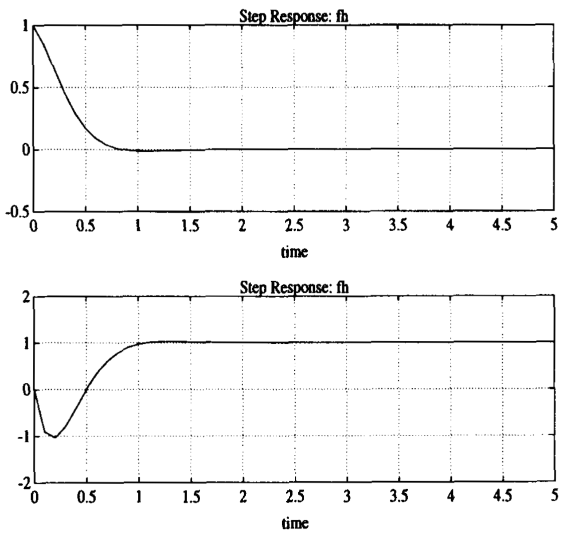


Fig. 19. Step response of master, free motion: $f_h - v_m$ and τ_{m1}.

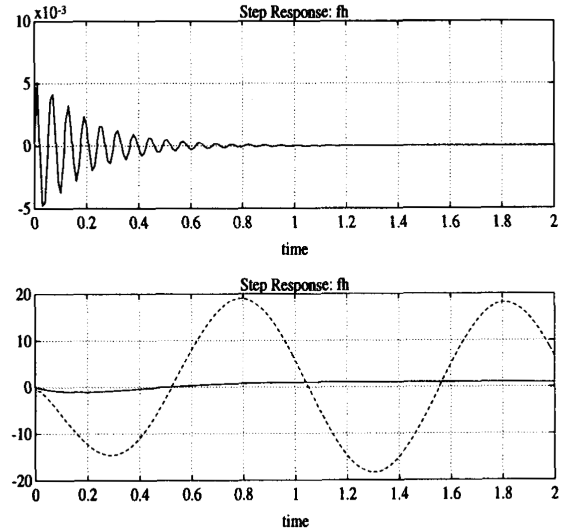


Fig. 20. Step response, free motion: $v_m - v_s$ and τ_{m1} and $\tau_s 1$.

9). The weighting matrices were chosen as

$$W_{m1}(s) = \alpha_{m1} \frac{s^2 + 10s + 100}{s^2 + 10s + 0.01}$$

for the weight on the error signal $(f_h - v_m)$,

$$W_{m2}(s) = \alpha_{m2} \frac{s}{0.001s + 1}$$

for the weight on the control signal τ_{m1} and

$$W_{m3}(s) = \alpha_{m3} \frac{s}{0.001s + 1}$$

to model the disturbance signal d_{m1}. The choice of the weighting function W_{m1} for the velocity error is based on the fact that the $\mathcal{H}_\infty$ design technique, in trying to make the norm of the operator from f_h to $W_{m1}(v_m - v_s)$ small, will tend to make the transfer function from f_h to $(v_m - v_s)$ resemble $W_{m1}^{-1}(s)$. Hence, $W_{m1}(s)$ was chosen accordingly. That is, given the standard second order system

$$z_1(s) = \left[1 - \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right] w_1(s)$$

the parameters ζ and ω_n were chosen to obtain the desired transient response for W_{m1}^{-1}. This design resulted in a fourth order, stable controller K_m. The closed-loop step response (i.e., $f_h(s) = \frac{1}{s}$) is illustrated in Fig. 19.

Next, we design the slave controller, K_s, for free motion as per Fig. 10. Note the equations for the inputs and outputs of the system are given in (3). The weighting matrices for the design were chosen as

$$W_{s1}(s) = \alpha_{s1} \frac{s^2 + 10s + 10000}{s^2 + 10s + 0.1}$$

for the weight on the error signal $(v_m - v_s)$,

$$W_{s2}(s) = \alpha_{s2} \frac{s}{s + 1000}$$

for the weight on the control signal τ_{s1} and

$$W_{s3}(s) = \alpha_{s3} \frac{s}{0.001s + 1}$$

to model the disturbance signal d_{s1}. This results in a stable controller. The closed-loop step response (i.e., $f_h(s) = \frac{1}{s}$) is illustrated in Fig. 20. This response (settling time < 1 second) is quite good considering the impulse response of the open loop plant has a settling time greater than 100 seconds.

B. Constrained Motion Design

For this design (see Fig. 16), the equations for the four signals z, w, y, and u are given by (5). The weighting matrices for the design are as follows:

$$W_1(s) = \alpha_1 \frac{s^2 + 18s + 10000}{s^2 + 18s + 0.01}$$

for the velocity error $(v_m - v_s)$,

$$W_2(s) = \alpha_2 \frac{s^2 + 10s + 10000}{s^2 + 10s + 0.1}$$

for the force reflection error $[f_s - (\tau_{m1} + \tau_{m2})]$, and

$$W_3(s) = \alpha_3 \frac{s}{0.1s + 1}$$

and

$$W_4(s) = \alpha_4 \frac{s}{0.1s + 1}$$

for the master and slave control torques (τ_{m2} and τ_{s2}). For

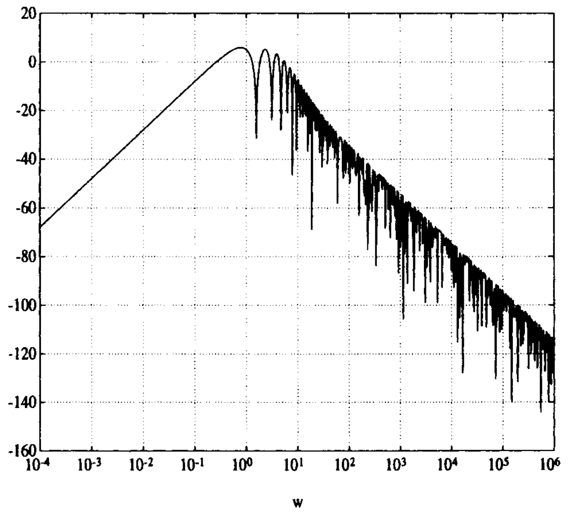


Fig. 21. Graph of $\Delta_4 G_{m1}$.

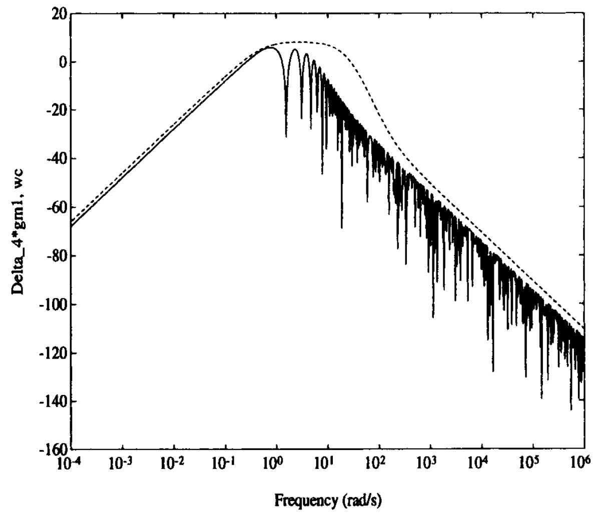


Fig. 22. Graph of $\Delta_4 G_{m1}\ W_c$.

our design, we let

$$Z_e = \frac{1000}{s}$$

represent the impedance of the slave manipulator's environment.

The function $W_c(s)$ must be designed so that (see (4))

$$|W_c j\omega| > |\Delta_\tau j\omega G_{m1} j\omega| \quad \forall\omega, \quad \forall\tau \le 4.$$

Suppose the maximum time delay incurred in the communication channel is 2 seconds in each direction (4 seconds total). Then let $\tau = 4$. Fig. 21 shows the graph of the function $|\Delta_4 j\omega G_{m1} j\omega|$. Let

$$W_c(s) = 5.0 \times \frac{s(0.007s+1)^2}{(0.035s+1)^3(1.9s+1)}.$$

Then,

$$|W_c j\omega| > |\Delta_4 j\omega G_{m1} j\omega| \quad \forall\omega$$

as illustrated in Fig. 22. In fact, it may be shown that this choice of $|W_c(j\omega)|$ is greater than $|\Delta_\tau(j\omega)G_{m1}(j\omega)|$ for all $\tau \le 4$, for all ω.

Using these weighting matrices, the system was designed via the μ-synthesis method, using μ-Tools in MATLAB, and the controller K was computed. The resulting closed loop step response (to input $f_h(s) = \frac{1}{s}$ for $\tau = 0$) of the error signals $(v_m - v_s)$ and $[f_h - (\tau_{m1} + \tau_{m2})]$ is shown in Fig. 23. Next, we check that the closed-loop system is stable for all time delays of $\tau \le 4$. That is, suppose we denote the transfer function from w_c to $(f_h - \tau_{m2})$ in Fig. 16 by $Q(s)$. Then, we require

$$\|\Delta_4 G_{m1} Q\|_\infty < 1$$

for stability of the system. The magnitude plot of $\Delta_4 G_{m1} Q$ is illustrated in Fig. 24. Note, from this figure, that we are within our specifications.

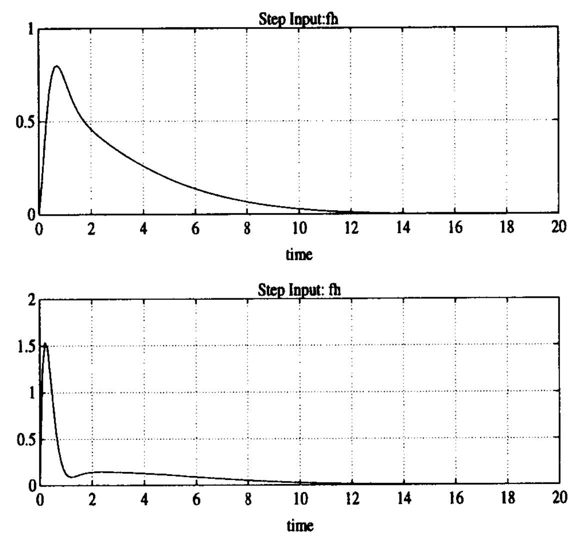


Fig. 23. Step response, constrained motion: $v_m - v_s$ and $[f_s - (\tau_{m1} + \tau_{m2})]$.

Using MATLAB, we now simulate the telerobotic system with a time delay of two seconds (each direction). Once again, we apply a step input, $f_h(s) = \frac{1}{s}$, to the system. The response is illustrated in Fig. 25. Note that although there is a total delay of four seconds from the initiation of hand controller input, f_h, to the time when force is "reflected back" to the master robot, the system is able to maintain stability. It can be shown that stability is achieved for all time delays of less than four seconds. That is

$$\|\Delta_\tau G_{m1} Q\|_\infty < 1 \quad \forall\tau \in [0, 4].$$

Now, suppose we choose a time delay greater than four seconds, say $\tau = 6$ seconds. This corresponds to a communication time delay of three seconds in each direction between

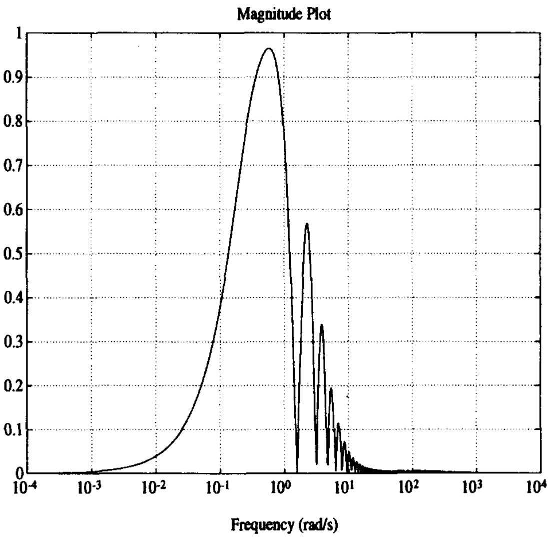


Fig. 24. Check for time delay stability margin: $|\Delta_4 G_{m1} Q|$.

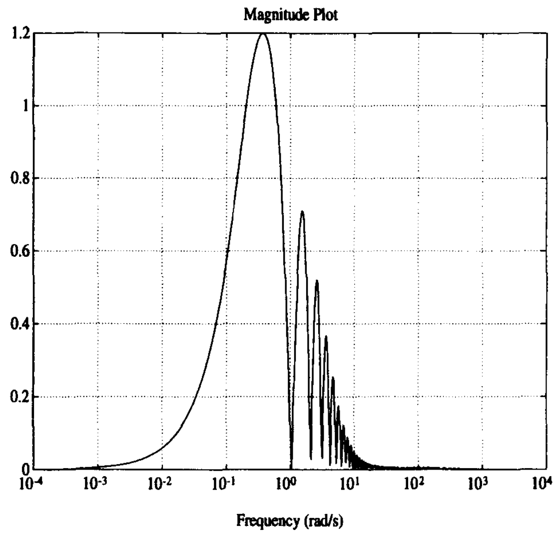


Fig. 26. Check for six-second time delay: $|\Delta_6 G_{m1} Q|$.

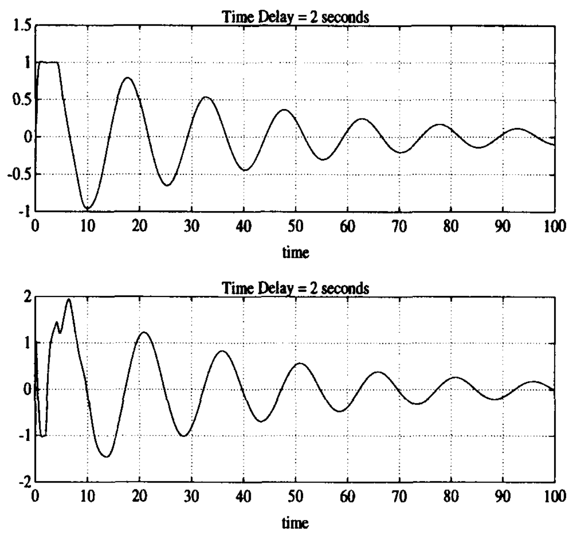


Fig. 25. Step response with $\tau = 2$: $v_m - v_s$ and $[f_s - (\tau_{m1} + \tau_{m2})]$.

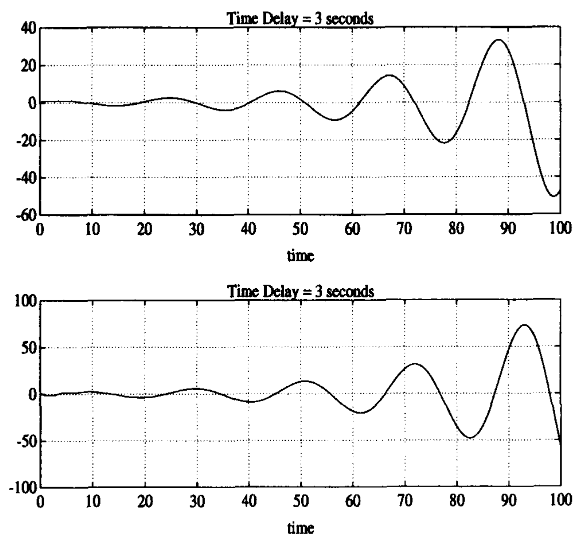


Fig. 27. Step response with $\tau = 3$: $v_m - v_s$ and $[f_s - (\tau_{m1} + \tau_{m2})]$.

the master and slave. Fig. 26 illustrates the magnitude plot of $\Delta_6 G_{m1} Q$. Note from this figure that

$$\|\Delta_6 G_{m1} Q\|_\infty > 1.$$

Hence, we cannot guarantee the system to be stable for a time delay of 3 seconds (in each direction) or greater through the communication channel. Using MATLAB, we now simulate the telerobotic system with a time delay of three seconds (each direction). Once again, we apply a step input to the system. The response is, in fact, unstable and is illustrated in Fig. 27. Thus, we can see from this example that our perturbation model is not overly conservative. That is, given an upper bound on the time delay, the design procedure will design for that time delay and only trade off that amount of time-delay stability margin against the performance criteria. It is interesting to note that the Anderson-Spong method would not be applicable here, since neither the master nor slave system is passive.

VI. Concluding Remarks

A systematic procedure has been presented for controller design for a bilateral force-reflecting teleoperator. The method requires no special assumptions, such as passivity of the master and slave systems. The system is designed to function both in free motion and when the slave is in contact with its environment, and also when there is a pre-specified upper bound on the communication time delay. The design is based on the $\mathcal{H}_\infty$-optimal control and μ-synthesis framework. The example shows that the method is not overly conservative.

REFERENCES

[1] C. Andriot, R. Fournier, and J. Vuillemey, "On the bilateral control of teleoperators with flexible joints and time delay by the passive approach," in *Proc. 5th Int. Conf. Advanced Robotics,* pp. 231–236, 1991.

[2] R. J. Anderson and M. W. Spong, "Bilateral control of teleoperators with time delay," in *Proceedings IEEE Conference on Decision and Control,* pp. 167–173, 1988.

[3] R. J. Anderson and M. W. Spong, "Asymptotic stability for force reflecting teleoperators with time delay," *IEEE Trans. Automat. Contr.,* pp. 1618–1625, 1988.

[4] G. J. Balas, J. C. Doyle, K. Glover, A. Packard, and R. Smith, *μ-Analysis and Synthesis Toolbox: User's Guide.* The Mathworks Inc., 1991.

[5] P. M. Bobgan and H. Kazerooni, "Achievable dynamic performance in telerobotic systems," in *Proc. IEEE Conf. Robotics Automat.,* pp. 2040–2046, 1991.

[6] T. L. Brooks, "Teleoperator system response for nuclear telepresence," STX Publication 90-03, ST Systems Corp., March 1990.

[7] J. C. Doyle, K. Glover, P. Khargonekar, and B. A. Francis, "State-space solutions to standard $calH_2$ and $calH_\infty$ control problems," *IEEE Trans. Automat. Contr.,* pp. 831–847, 1989.

[8] W. R. Ferrell, "Remote manipulation with transmission delay," *IEEE Trans. Human Factors in Electronics,* pp. 24–32, 1965.

[9] B. A. Francis, *A Course in $\mathcal{H}_\infty$ Control Theory.* New York: Springer-Verlag, 1987.

[10] A. A. Goldenberg and D. Bastas, "On the bilateral control of force reflecting teleoperation," in *Proc. World Congress of IFAC,* Tallin, USSR, pp. 215–220, 1990.

[11] A. A. Goldenberg, D. Bastas, and Y. Strassberg, "On the bilateral control of master-slave teleoperators," *Robotersysteme,* pp. 91–99, 1991.

[12] B. Hannaford, "A design framework for teleoperators with kinesthetic feedback," *IEEE Trans. Robotics Automat.,* pp. 426–434, 1989.

[13] B. Hannaford and P. Fiorini, "A detailed model of bi-lateral teleoperation," in *Proc. Int. Conf. Syst. Man Cyber.,* pp. 117–120, 1988.

[14] H. Kazerooni, T. I. Tsay, K. Hollerbach, "A controller design framework for telerobotic systems," *IEEE Trans. Contr. Syst. Tech.,* pp. 50–62, 1993.

[15] G. M. H. Leung and B. A. Francis, "Bilateral control of teleoperators with time delay through a digital communication channel," in *Proc. Thirtieth Ann. Allerton Conf. Commun., Contr., Comput.,* pp. 692–701, 1992.

[16] G. Niemeyer and J. E. Slotine, "Stable adaptive teleoperation," in *Proc. Am. Contr. Conf.,* pp. 1186–1191, 1990.

[17] R. Ravindran and K. H. Doetsch, "Design aspects of the shuttle remote manipulator control," in *AIAA Guidance Contr. Conf.,* 1982.

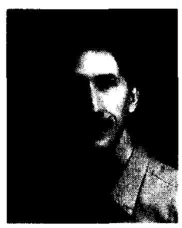
[18] T. B. Sheridan, "Telerobotics," *Automatica,* pp. 487–507, 1989.

[19] T.I. Tsay and H. Kazerooni, "A design framework for telerobotics using the H_∞ approach," in *Proc. Am. Contr. Conf.,* pp. 2931–2935, 1992.

[20] J. Vertut, A. Micaelli, P. Marchal, and J. Guittet, "Short transmission delay on a force reflective bilateral manipulator," in *Proc. 4th Rom-An-Sy,* 1981, pp. 269.

Gary Man Hay Leung received the B.A.Sc., M.A.Sc., and Ph.D. degrees in electrical engineering from the University of Toronto in 1988, 1990, and 1994, respectively.

He is currently a systems analyst at AlliedSignal Aerospace Canada in Toronto, Canada.



Bruce A. Francis (F'88) received the B.A.Sc. and M.Eng. degrees in mechanical engineering and the Ph.D. degree in electrical engineering from the University of Toronto in 1969, 1970, and 1975, respectively.

He has held research and teaching positions at the University of California at Berkeley, the University of Cambridge, Concordia University, McGill University, Yale University, the University of Waterloo, Caltech, and the University of Minnesota. He is currently a Professor of Electrical and Computer Engineering at the University of Toronto where he does research in control theory and applications.

Jacob Apkarian received the B.Eng. from McGill University in 1980 and the M.A.Sc. and Ph.D. degrees from the University of Toronto in 1984 and 1989, respectively, all in electrical engineering.

He has held the positions of Assistant Professor at the University of British Columbia, Senior Researcher in robotics for persons with physical disabilities at Lyndhurst Hospital in Toronto, and Senior Controls Engineer at Spar Aerospace in Toronto. He is currently President of Quanser Consulting Inc., a company that designs and manufactures experimental hardware for control system education and research. He is also an Adjunct Professor in the Institute of Biomedical Engineering at the University of Toronto.